# THE LOW ENERGY SPECTRUM OF COSMIC RAYS AS AN INDICATOR OF PRIMARY SOURCE CHARACTERISTICS AND INTERSTELLAR PROPAGATION

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## ABSTRACT

The low energy spectra of protons and helium nuclei, recently measured by McDonald and Ludwig, and by Fan, Gloeckler and Simpson, are here corrected for solar modulation and diffusive passage through interstellar matter. The resulting source spectrum is exhibited and compared with the spectrum of accelerated particles inferred from high energy data. This analysis yields a transmission efficiency of the source environment which is similar in form to the velocity dependent solar modulation of low energy particles.

# The Low Energy Spectrum of Cosmic Rays As An Indicator of Primary Source Characteristics and Interstellar Propagation.

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<u>Introduction:</u> In the study of galactic cosmic radiation it is of interest to divide the evolution of the observed particles into four stages:[1]

(a) injection from source regions

(b) possible partial confinement in source regions

(c) motion in the interstellar medium

(d) solar modulation.

Interstellar propagation has been studied extensively by many workers[2,3,4] from a detailed determination of the charge spectrum of the cosmic radiation. In particular, it has been found that Li, Be, B (called L nuclei) occur in primary cosmic radiation with an abundance very large compared to their universal abundance. As these nuclei get easily destroyed in nuclear interactions at stellar temperatures, it is reasonable to assume that they are probably not present in the source regions, but are produced in nuclear collisions of heavier nuclei with the protons of the

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interstellar medium. Using the abundance of these light nuclei relative to heavier nuclei, an estimate of the mean amount of matter traversed in the interstellar medium[5] has been made. For relativistic energies, a recent estimate,[6] taking into account effects due to decay of unstable isotopes formed in collisions and other details of nuclear collisions, gives a value of 2.5 $gm/cm^2$ as the mean amount of matter traversed by cosmic rays in the interstellar medium and is considered to be a reliable measure of this quantity.[7] At low energies, however, the picture is not so clear and some experiments[8,9] seem to indicate an increase in the ratio of the abundance of L nuclei to that of heavier nuclei. Kaplon and Skadron[1] have interpreted the increase of this ratio as evidence that the traversal of matter by low energy nuclei is enhanced by a preferential confinement for low energy particles inside source regions that are surrounded by partially reflecting boundaries.

The analysis presented in this paper is based primarily upon recent experiments, conducted on board the IMP-I satellite, which have yielded the first clear measurement of the low energy spectrum of cosmic rays. The proton spectrum thus obtained by McDonald and Ludwig[10] and the helium nucleus spectrum obtained by Fan, Gloeckler & Simpson[11] are found to exhibit pertinent information about source transmission and interstellar propagation of low energy particles. Since these particles lie in the interval 20 - 100 MeV per nucleon, ionization loss is the dominant feature of their propagation in the interstellar medium, and their energy is thereby greatly modified. This degradation of energy is a direct sensitive measure of the quantity of matter traversed by cosmic rays. Due to their low magnetic rigidity, these particles also serve as a probe of the magnetic structure that is characteristic of the source environment.

In this paper we define the source spectrum in an operational way. We first take the experimentally observed energy spectrum and "demodulate" it for solar cycle effects. This "demodulated" spectrum, which represents the cosmic ray energy spectrum outside the solar system, is traced back through the interstellar medium by taking into account the energy loss due to ionization. The dispersion in paths arising from the diffusion of cosmic rays in the interstellar medium is considered in the analysis; this involves a specification of the distribution in path lengths about the mean value of 2.5 gms/cm$^2$. Finally, by comparing the source spectrum obtained in this way with the spectrum of accelerated particles inferred from high energy data, we are able to reconstruct the transmission function of the physical source environment.

The Low Energy Proton and Helium Nucleus Spectrum:

Figure 1 exhibits the proton and helium nucleus rigidity spectra obtained from recent balloon and IMP-I satellite measurements. The helium nucleus data have been normalized to the proton data by employing a scale factor of 7. Although the proton and helium nucleus rigidity spectra are comparable at high rigidities, it is clear that at rigidities below $\sim$ 1500 MV the two spectra are definitely distinct. This shows that rigidity can not be used as a universal parameter for characterizing the spectrum of cosmic ray particles.

Figure 2 exhibits the proton and helium nucleus spectra in a representation based upon the kinetic energy per nucleon. In this instance, the helium nucleus data have been scaled by 5.7 to achieve a normalization to the proton data. As indicated in Figure 2, a single curve describes all the pertinent proton and helium nucleus data points. This suggests that the kinetic energy per nucleon, or any other purely velocity dependent function, might be used to completely characterize the spectrum of all cosmic ray particles. The analytical expression for the indicated curve is

$$\frac{dJ}{dE} = 10^{8} E^{1.5} (E + 500)^{-4} \tag{I}$$

where $dJ/dE$ has the units of particles /m$^2$-sec-ster-MeV, and E is the kinetic energy per nucleon in MeV. This analytical representation of the observed spectrum is shown again as curve A in Figure 3. At high energy (i.e. for $E \gg 500$ MeV per nucleon) this expression meets the asymptotic constraint of the high energy data[12], viz:

$$\frac{dJ}{dE} \propto E^{-2.5} \tag{2}$$

For protons and helium nuclei in the low energy range the important effects to be taken into account are ionization loss in the interstellar medium and solar modulation. At low energy these processes affect the protons and helium nuclei in an identical way as regards changes per nucleon. The reasons for this are that I) for equal $Z^2 / A$, which is the case for protons and helium nuclei, the ionization loss per nucleon is a universal function of velocity, and 2) the low energy solar modulation is a universal function of velocity.

McDonald and Ludwig[10] have deduced the galactic proton spectrum expected at Earth under the assumption of:
i) an energy spectrum at injection of the form

$$\mu(\mathcal{E}) = \frac{4\pi \times 10^{-4}}{3} \Big/ \beta (1+\mathcal{E})^{2.5} \quad \text{protons/m}^3\text{-BeV} \tag{3}$$

where $\mu(\mathcal{E})$ is the density of cosmic rays of kinetic energy $\mathcal{E}$, in BeV, and $\beta$ is the proton velocity in units of the velocity of light, ii) 2.5 gm/cm$^2$ of hydrogen traversed in the interstellar medium, and iii) a solar modulation given by

$$(dJ/dE) = (dJ/d\mathcal{E})_0 \exp(-0.8/\beta) \tag{4}$$

where $(dJ/d\mathcal{E})$ and $(dJ/d\mathcal{E})_0$ are the modulated and unmodulated spectra, respectively.

The results of these calculations by McDonald and Ludwig are shown by the dotted curve of Figure 1. By comparing their proton data with this dotted curve, they conclude that either the source spectrum is much steeper than what has been assumed or the solar modulation is weaker at the low energies experimentally observed. Using the same solar modulation, the present analysis indicates that the source spectrum at low energies is strongly affected by the transmission characteristics of the source environment.

Solar Modulation:

Parker[13] has described the 11 year solar modulation of cosmic rays in terms of a quasi-stationary solution to a Smoluchowski generalized diffusion equation[14] for the process of a charged particle diffusing among the magnetic kinks convected away from the sun by the plasma wind. For energies sufficiently low such that the Larmor radius (R) is small compared with the linear dimension ($\ell$) of the magnetic kinks, the solution may be expressed as

$$N_{IN} = (N_{OUT}) \exp[-K/\beta] \qquad (5)$$

where $N_{IN}$ is the cosmic ray density at the orbit of Earth, $N_{OUT}$ is the cosmic ray density outside the solar environment, and $K$ is a solar modulation parameter that is proportional to the solar wind velocity and the thickness of the solar cavity, measured in units of kink intervals.

For energies high enough such that $R > \ell$ the solution takes on the form

$$\mu_{IN} = (\mu_{OUT}) \exp\left[-(K/\beta^3)(1-\beta^2)\left(\frac{\ell Z}{R_o A}\right)^2\right] \tag{6}$$

where $R_o = Mc^2/eB$, M is the proton mass, B is the magnetic field, and Z and A are the particle charge and mass number respectively.

Recent work on proton and helium nucleus spectra near solar minimum suggests that solar modulation effects are essentially velocity dependent.[15] The fact that protons and helium nuclei have the same spectra, in energy per nucleon, indicates that the dominant modulation is a function of velocity alone. For the present analysis, the change-over from the purely velocity dependent function (5) for low energies to the formula (6) for high energies is effected at a kinetic energy per nucleon ($E_o$) given by

$$E_o = Mc^2\left[\left(1+\left\{\frac{\ell Z}{R_o A}\right\}^2\right)^{1/2} - 1\right]. \tag{7}$$

Figure 3 shows the demodulated spectrum, curve B, thus obtained from the observed spectrum, curve A, for $E_o$ = 1 BeV. Below ~ 500 MeV per nucleon, curve B exhibits the characteristic of a flat spectrum. We have noted that curve B remains essentially the same for all $(\ell Z/R_o A) > 1$. Therefore we may fit the data for helium nuclei, where A = 2Z, as well as the data for protons, where A = Z, by requiring that $\ell/R_o > 2$. This may be achieved with reasonable values for $\ell$ and B (e.g. $\ell \sim 10^{12}$ cm., $B \sim 10^{-5}$ gauss).

Extrapolation to the Source:

By a detailed consideration of ionization losses, the solar demodulated spectrum is extrapolated to the source. Since these particles diffuse through the interstellar medium, the traversal of matter will however exhibit a dispersion about the mean value of 2.5 gm/cm$^2$, and as these energies are low, the inferred source energy of each observed particle will depend critically on the actual path traversed. In the appendix an expression for the distribution of path lengths due to diffusion is derived. The variance of the path lengths associated with this distribution is ( $\sqrt{2/5}$ ) of the mean path length, and therefore this complication to the extrapolation clearly cannot be ignored.

Each observed particle is, in principle, mapped into a unit modulus energy distribution at the source according to the a priori distribution in path lengths arising from a diffusion process. These individual distributions at the source are weighted by the solar demodulation function, (5) and (6), and the resulting source spectrum, curve C, is shown in Figure 3. This source spectrum exhibits a peak at about 100 MeV per nucleon and falls rapidly at lower energies.

Discussion:

When compared with the observed spectrum, the source spectrum shows the systematic effects of energy loss by ionization, particularly at the low energy end. The smoothness at the low energy end is due to the dispersion in path lengths in the interstellar medium. If one were to assume that all particles traverse exactly 2.5 gm/cm$^2$, then the source spectrum thereby inferred would exhibit an abrupt cutoff at about 80 MeV per nucleon.

If one assumes that high energy particles ( > several BeV) can freely propagate away from the source accelerator and are not modulated drastically by galactic or solar magnetic fields, then the energy spectrum of accelerated particles, at high energies, is the same as the asymptotic form (2) of the observed spectrum, viz:

$$\frac{dJ}{dE} = 10^{8} E^{-2.5}. \qquad (8)$$

This is shown as curve D in Figure 3.

This spectrum (8) for accelerated particles probably does not remain valid down to extremely low energies, but we know that it is an adequate description for several decades in the region of several BeV. As pointed out by Syrovatskii[16] this form (8) of the spectrum corresponds to a general situation of energy equipartition among cosmic rays, magnetic fields, and turbulent motion. If we assume that the same form of this spectrum is valid down to about 100 MeV, then the ratio of the source intensity, curve C, to the intensity given by curve D is a measure of the transmission efficency of the source environment for the energy per nucleon under consideration. Figure 4 shows this transmission coefficient as a function of the reciprocal velocity. The fact that this coefficient exhibits a straight line in a semi-log plot with respect to $1/\beta$ suggests that the accelerated source particles remain partially confined by a process of diffusion. The cosmic ray density "outside the source" is thereby related to the density "inside the source" by a simple relation of the form

$$\mu_{OUT}(E) \propto [\mu_{IN}(E)] \exp(-K'/\beta) \qquad (9)$$

where k' is the slope of the transmission ratio plot (Fig. 4).

Since the cosmic ray source spectrum appears to be a universal function of velocity rather than rigidity, we infer that the confinement to the source environment does not arise from trapping by ordered magnetic fields. Rather, a comparison of (9) with (5) indicates that the process involved here is one of diffusion, similar to that prevailing in the solar system. This suggests that the magnetic environment of the physical source of cosmic rays might be similar in form to the solar magnetic environment (e. g. magnetic turbulence convected outward by a plasma wind).

Acknowledgements:

It is a pleasure to acknowledge the encouragement of Frank B. McDonald and the mathematical counsel of Philip B. Abraham.

## Appendix:

The dispersion in cosmic ray particle paths from the source to the solar system is here attributed to a diffusion mechanism. For a particle produced at the space-time origin of a homogenous isotropic diffusive medium, the probability per unit time of first passage beyond a radius (a) is given by[14]

$$P(t) = \sum_{n=1}^{\infty} 2 \left[\frac{n\pi}{a}\right]^2 D \exp\left[i(n+1)\pi - \left(\frac{n\pi}{a}\right)^2 D t\right] \qquad \text{(A1)}$$

where D is the diffusion constant characteristic of the medium.

The mean time of first passage $\langle t \rangle$ is constructed as

$$\langle t \rangle = \int_0^{\infty} t P(t)\, dt = a^2/(6D) \qquad \text{(A2)}$$

The second temporal moment $\langle t^2 \rangle$ is constructed as

$$\langle t^2 \rangle = \int_0^{\infty} t^2 P(t)\, dt = \left(\frac{7}{180}\right)\left(a^2/D\right)^2. \qquad \text{(A3)}$$

The variance $(\sigma_t)^2$ of the first passage time is then given by

$$(\sigma_t)^2 = \langle (t - \langle t \rangle)^2 \rangle = \langle t^2 \rangle - (\langle t \rangle)^2 = \frac{1}{90}\left(\frac{a^2}{D}\right)^2. \qquad \text{(A4)}$$

By comparing the mean first passage time $\langle t \rangle$, as given by (A2), with the variance, as given by (A4), we note the relation

$$\sigma_t = \sqrt{\frac{2}{5}}\, \langle t \rangle, \qquad \text{(A5)}$$

If we assume that the quantity of matter traversed (S) by an observed particle is proportional to its total transit time, source to observer, then we can use the relation given by (A5) to infer that

$$\sigma_s \equiv \sqrt{\langle (S - \langle S \rangle)^2 \rangle} = \sqrt{\tfrac{2}{5}} \; \langle S \rangle . \qquad \text{(A6)}$$

We consider a sample of observed cosmic ray particles which is characterized by a certain value for $\langle S \rangle$ (e.g. 2.5 gm/cm$^2$) and assign the corresponding variance (A6) prescribed by diffusion theory. The well defined variance and mean here ascribed to (S) are sufficient to construct a normal gaussion distribution function, viz:

$$P(s) = \frac{\exp\left[-\dfrac{(S - \langle s \rangle)^2}{2(\sigma_s)^2}\right]}{(\sqrt{2\pi})\,\sigma_s} . \qquad \text{(A7)}$$

Insertion of the explicit expression (A6) of the variance into (A7) yields

$$P(s) = \frac{\sqrt{5/\pi}}{2\langle s \rangle} \exp\left[-\frac{5}{4}\left(\frac{S - \langle S \rangle}{\langle s \rangle}\right)^2\right] , \qquad \text{(A8)}$$

This distribution function (A8) is renormalized to account for the exclusion of negative values for (S), as follows:

$$P' = P \Big/ \left(\int_0^\infty P \, ds\right) \qquad \text{(A9)}$$

where $\int_0^\infty P\,ds = 0.9429.$

This normalization procedure (A9) is based upon the approximation that

$$\langle S \rangle \gg \text{(shortest s, source to observer).} \qquad \text{(A10)}$$

This assumption (A10) is valid in practice since the material diameter of the entire galaxy is an order of magnitude less than $\langle S \rangle$.

The final expression of the renormalized distribution function utilized here is:

$$P'(S) = \left(\frac{\sqrt{5/\pi}}{1.8858}\right) \frac{\exp.\left[-\frac{5}{4}\left(\frac{S-\langle S\rangle}{\langle S\rangle}\right)^2\right]}{\langle S\rangle} \qquad \text{A11}$$

where:

$$\int_0^\infty P' \, dS = 1.0000.$$

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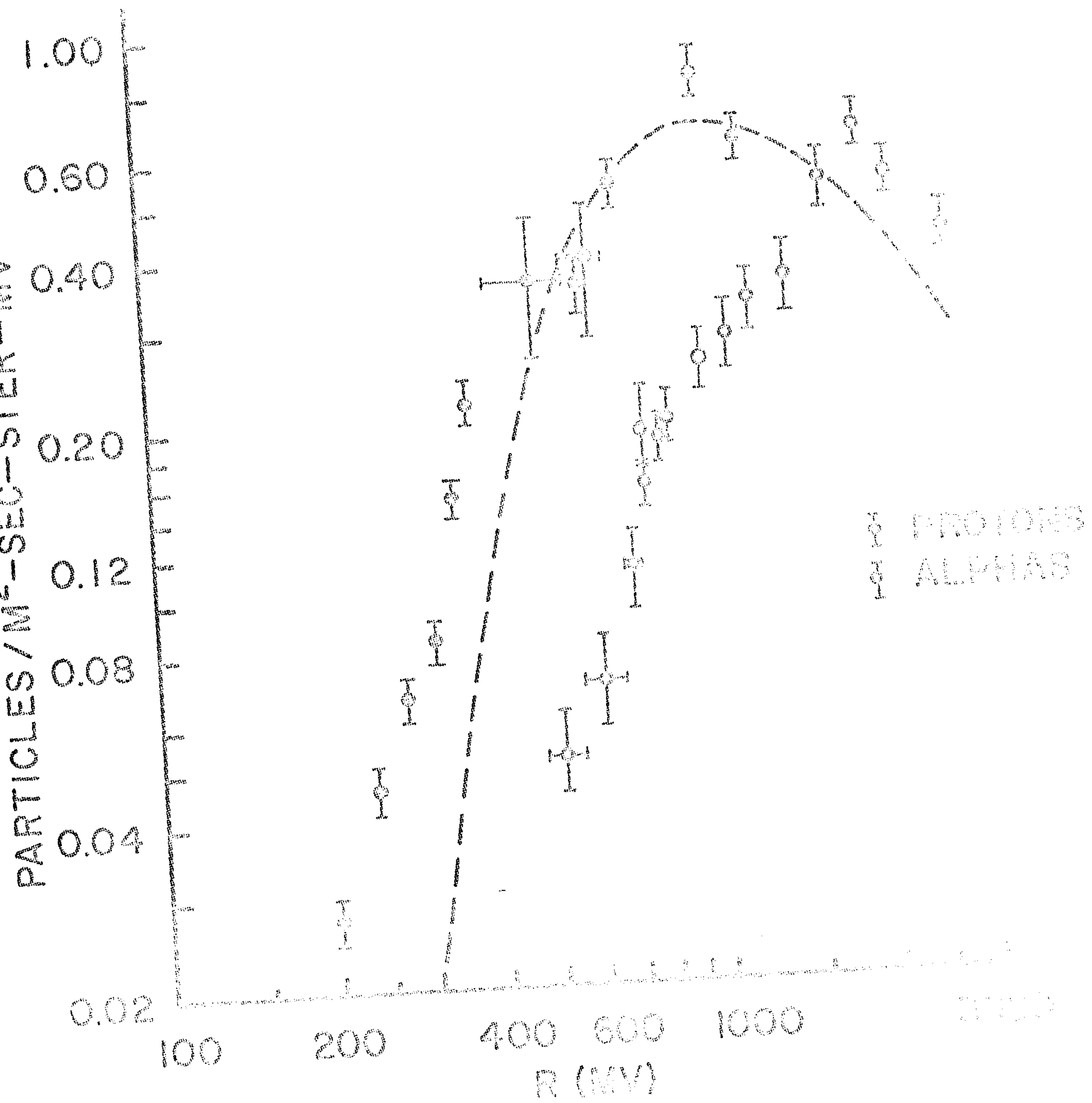
## FIGURE CAPTIONS

Fig. 1 Proton and He nucleus rigidity spectra. The low rigidity data were obtained from the IMP-1 satellite during the period November 1963-May 1964 (references 10,11). The other data were obtained from balloon measurements during June 1963 (references 17,18). The dotted curve was calculated by McDonald and Ludwig (reference 10).

Fig. 2 The proton and He nucleus data (reference 10,11, 17, 18) plotted in a kinetic energy per nucleon representation. The solid line is equation (1).

Fig. 3 The energy per nucleon spectra as observed (A), solar demodulated (B), corrected for ionization (C), and extrapolated from higher energies, for comparison (D).

Fig. 4 Source transmission coefficient plotted as a function of $1/\ell$. Some representative calculated points are shown. The slope of the straight line is -4.[illegible]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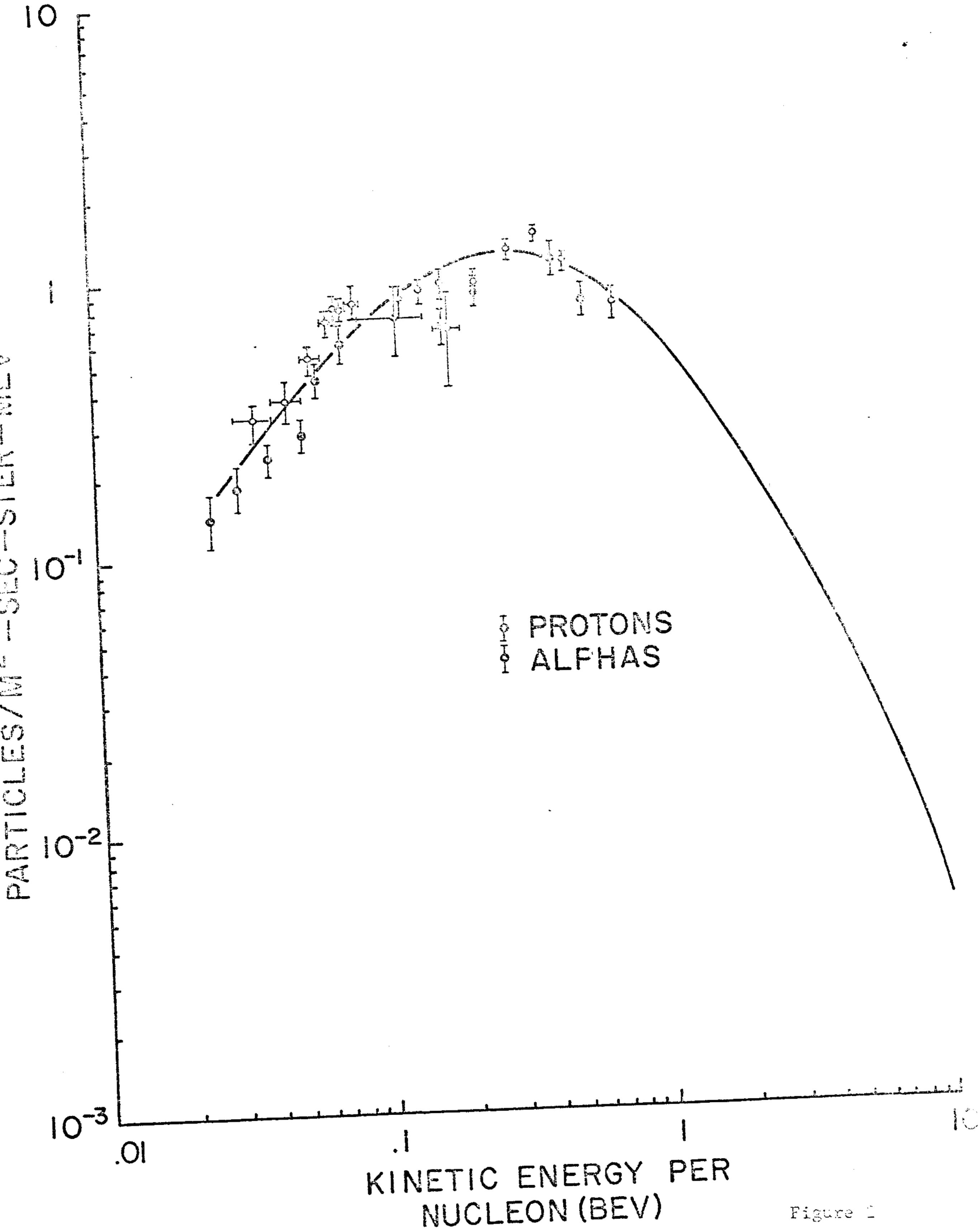
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KINETIC ENERGY PER NUCLEON (BEV)

Figure 2

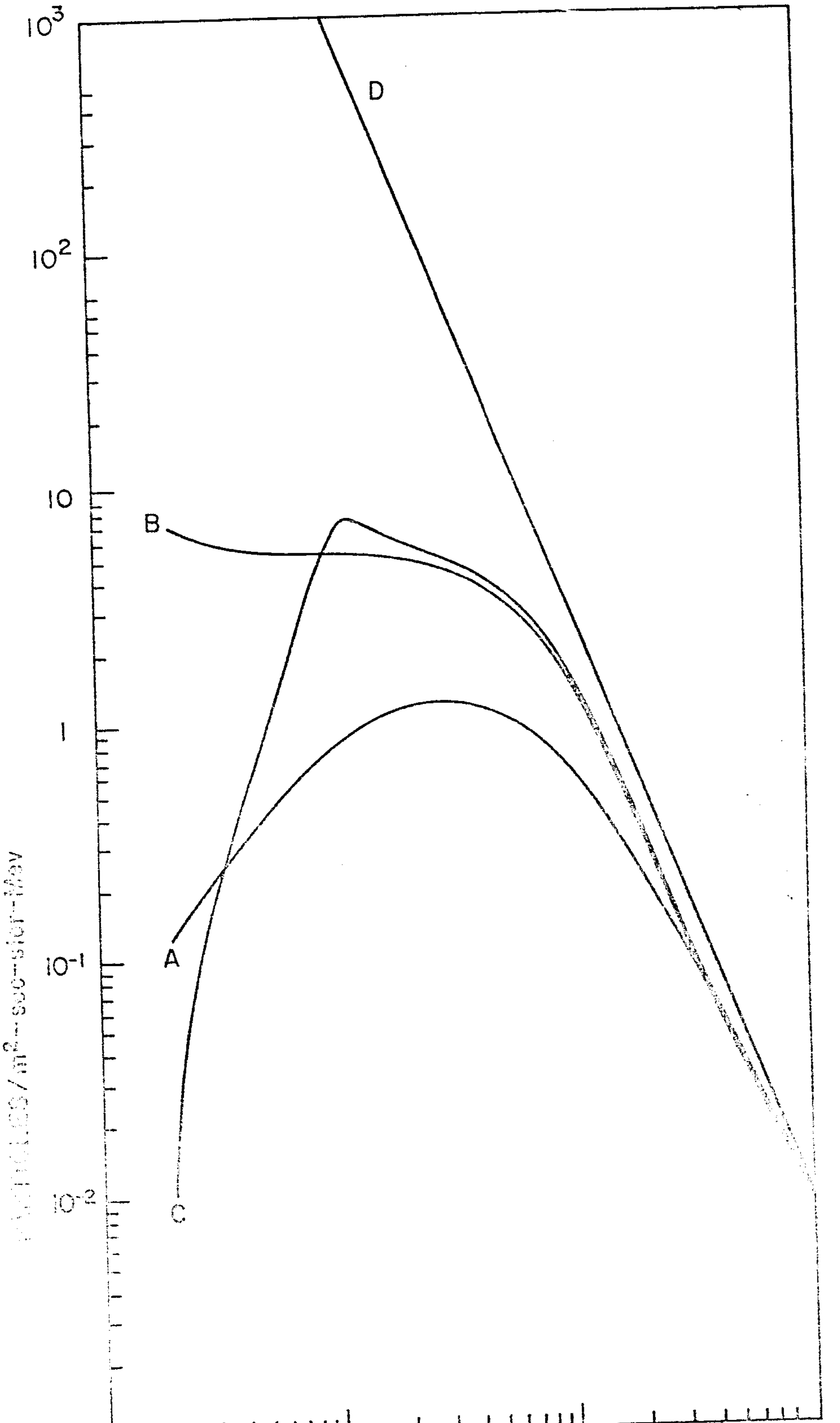
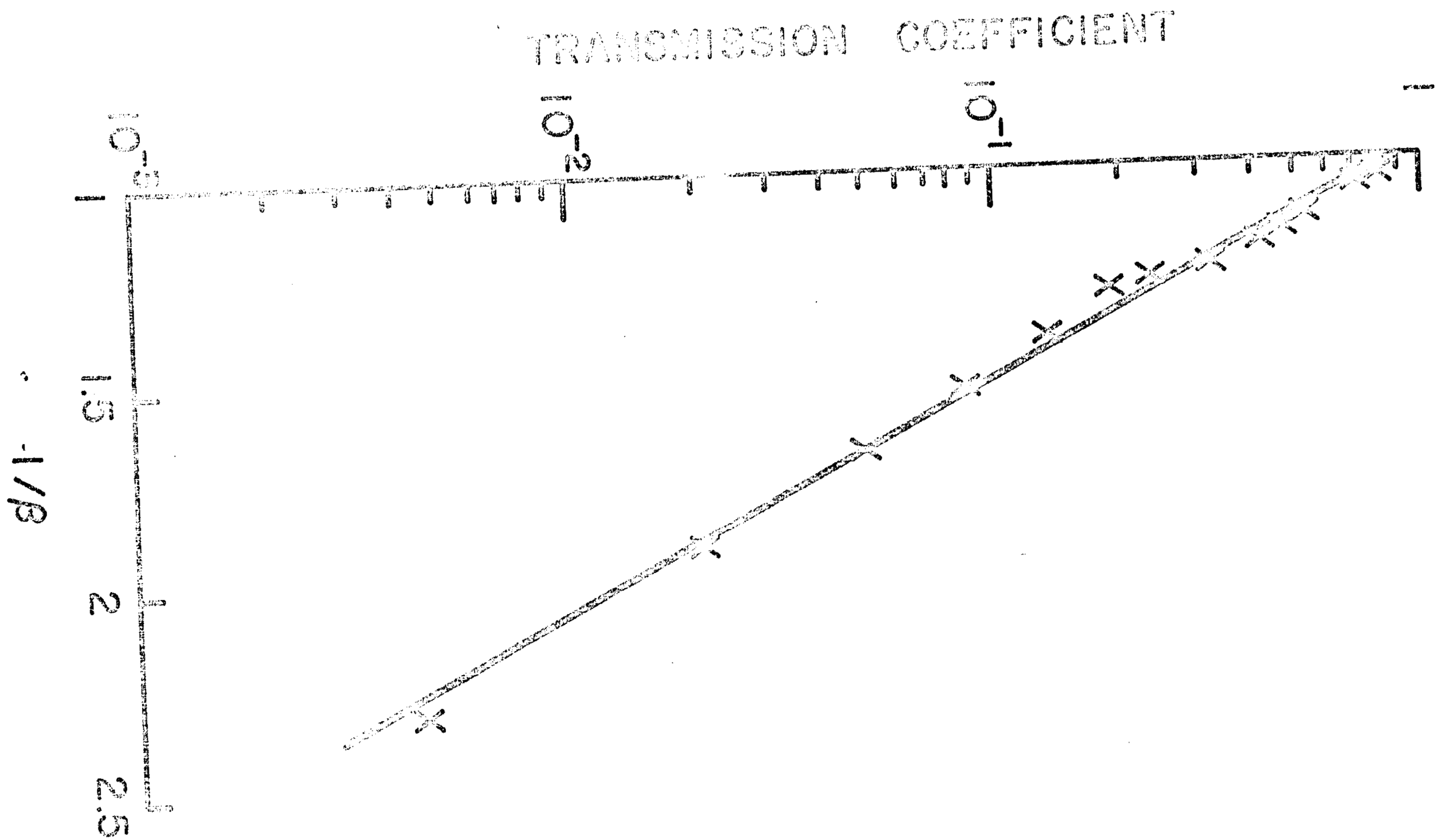



Figure 3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